Homework 5 Solutions

Nuclear Engineering 162

Due 5 March 2008

V.1 Turner 7.9 (Page 171)
From Fig. 5.6 find LET_∞ for a
(a) 2 MeV alpha particle.
1.8e3 MeV cm^{-1}
(b) 100 MeV muon.
2.3 MeV cm^{-1}
(c) 100 keV positron.
4.3 MeV cm^{-1}

V.2 Turner 7.12 (Page 171)
In section 7.4 we calculate the specific ionization of a 5 MeV alpha particle in air ($3.42 * 10^4 \text{cm}^{-1}$) and in water ($4.32 * 10^7 \text{cm}^{-1}$). Why is the specific ionization so much greater in water?

The stopping power in a medium is directly proportional to the number of electrons per unit volume in the medium and roughly inversely proportional to the ionization energy so the specific ioniztion in water for alpha particles is much greater than that in air.

V.3 Turner 7.14 (Page 171)
(a) Define energy straggling.
The phenomenon of unequal energy losses of particles in a medium under identical conditions is called energy straggling. (see page 161)
(b) Does energy straggling cause range straggling?
Yes.

V.4 Turner 9.3 (Page 235)
(a) Estimate the average energy that a 2-MeV neutron transfers to a deuteron in a single collision.
(b) What is the maximum possible energy transfer?
The maximum energy is given by:

$$Q_{max} = \frac{4mME_n}{(M+m)^2}$$

$$Q_{max} = \frac{4 \cdot 1 \cdot 2 \cdot 2\text{MeV}}{(1+2)^2}$$

$$Q_{max} = 1.78\text{MeV}$$

The average energy is given by:

$$Q_{avg} = \frac{1}{2} Q_{max}$$

$$Q_{avg} = \frac{1}{2} \cdot 1.78\text{MeV}$$

$$Q_{avg} = 0.89\text{MeV}$$

V.5 Turner 9.8 (Page 236)

(a) Estimate the average recoil energy of a carbon nucleus scattered elastically by 1-MeV neutrons.
The maximum energy is given by:

$$Q_{max} = \frac{4mME_n}{(M+m)^2}$$

$$Q_{max} = \frac{4 \cdot 1 \cdot 12 \cdot 1\text{MeV}}{(1+12)^2}$$

$$Q_{max} = 0.284\text{MeV}$$

The average energy is given by:

$$Q_{avg} = \frac{1}{2}Q_{max}$$

$$Q_{avg} = \frac{1}{2} \cdot 0.284\text{MeV}$$

$$Q_{avg} = 0.142\text{MeV}$$

(b) What is the average recoil energy of a hydrogen nucleus?

$$Q_{max} = \frac{4mME_n}{(M+m)^2}$$

$$Q_{max} = \frac{4 \cdot 1 \cdot 1 \cdot 1\text{MeV}}{(1+1)^2}$$

$$Q_{max} = 1\text{MeV}$$

The average energy is given by:

$$Q_{avg} = \frac{1}{2}Q_{max}$$

$$Q_{avg} = \frac{1}{2} \cdot 1\text{MeV}$$

$$Q_{avg} = 0.5\text{MeV}$$

(c) Discuss the relative importance of these two reactions as a basis for producing biological effects in soft tissue exposed to 1-MeV neutrons.
About 86 weight percent of soft tissue is water, so the neutron-hydrogen scattering is more important than the neutron-carbon scattering. For 100g of soft tissue:

$$\frac{86\text{g}H_2O}{16\text{gmol}} \cdot \frac{\text{mol}H}{\text{mol}H_2O} = 10.75\text{mol}H$$

$$\frac{11\text{g}C}{12\text{gmol}} = 0.92\text{mol}C$$

Moreover, the Q_{avg} for carbon is about one third of that for hydrogen so the neutron-carbon scattering becomes comparatively insignificant.

V.6 Turner 9.20 (Page 237)

The thermal-neutron reaction $^{15}\text{B}(n,\alpha)^3\text{Li}$ leaves the ^3Li nucleus in the ground state 7% of the time. Otherwise, the reaction leaves the ^3Li nucleus in an excited state, from which it decays to the ground state by emission of a 0.48-MeV gamma ray.

(a) Calculate the Q value of the reaction in both cases.
For the transition to the ground state:

$$Q = (\Delta_b + \Delta_n - \Delta_{Li} - \Delta_\alpha) = 12.052 + 8.0714 - 14.907 - 2.4248\text{MeV}$$
$$Q_{ground} = 2.7916\text{MeV}$$

For the transition to the excited state:

$$Q = (\Delta_b + \Delta_n - \Delta_{Li} - \Delta_\alpha - \Delta_\gamma) = 12.052 + 8.0714 - 14.907 - 2.4248 - 0.48\text{MeV}$$
$$Q_{ground} = 2.3116\text{MeV}$$

(b) Calculate the alpha-particle energy in both cases.

$$E = \frac{M}{M+m}Q$$

For the transition to the ground state:

$$E = \frac{7}{7+4}2.7916\text{MeV}$$
$$E = 1.78\text{MeV}$$

For the transition to the excited state:

$$E = \frac{7}{7+4}2.3116\text{MeV}$$
$$E = 1.47\text{MeV}$$

V.7 Turner 9.27 (Page 237)
A sample containing 127 g of ^{23}Na (100% abundant) is exposed to a beam of thermal neutrons at a constant fluence rate of $(1.19 \cdot 10^4\text{cm}^{-2}\text{s}^{-1})$. The thermal-neutron capture cross section for the reaction $^{23}\text{Na}(n,\gamma)^{24}\text{Na}$ is 0.53 barn.
(a) Calculate the saturation activity.

$$N_T = \frac{mN_A}{M} = \frac{127\text{g} \cdot 6.02 \cdot 10^{23}\text{atoms/mol}}{23\text{g/mol}} = 3.325 \cdot 10^{24}\text{atoms}$$
$$SA = \Phi\sigma N_T = (1.19 \cdot 10^4\text{n/cm}^2\text{s})(0.53 \cdot 10^{-24}\text{cm}^2)(3.325 \cdot 10^{24}\text{atoms})$$
$$SA = 2.1 \cdot 10^4\text{Bq}$$

(b) Calculate the ^{24}Na activity in the sample 24 h after it is placed in the beam.

$$\lambda = \frac{ln2}{t_{1/2}} = \frac{ln2}{15\text{h}} = 0.0462\text{h}^{-1}$$
$$A = \lambda N = \Phi\sigma N_T\left(1 - e^{-\lambda t}\right) = 2.1 \cdot 10^4\text{Bq}\left(1 - e^{-0.0462\text{h}^{-1}24\text{h}}\right)$$
$$A = 1.41 \cdot 10^4\text{Bq}$$

(c) How many ^{23}Na atoms are consumed in the first 24 h?

$$t = 24\text{h} = 8.64 \cdot 10^4\text{s}$$

^{23}Na atoms lost $= \Phi\sigma N_T t = \left(2.1 \cdot 10^4\text{Bq}\right)\left(8.64 \cdot 10^4\text{s}\right)$
^{23}Na atoms lost $= 1.81 \cdot 10^9$ atoms.

V.8 Turner 9.30 (Page 238)
To make a ^{60}Co source, a 50-g sample of cobalt metal (^{59}Co, 100% abundant) is exposed to thermal neutrons at a constant fluence rate of $(10^9\text{cm}^{-2}\text{s}^{-1})$. The thermal-neutron capture cross section is 37 barns.
(a) How much exposure time is required to make a 1-mCi source of ^{60}Co?

$$\lambda = \frac{ln2}{5.271\text{y} \cdot 365\text{d/y} \cdot 24\text{h/d} \cdot 60^2\text{s/h}} = 4.17 \cdot 10^{-9}\text{s}^{-1}$$

$$\sigma_\gamma = 37 \cdot 10^{-24}\text{cm}^2$$

$$A = 1\text{mCi} = 3.7 \cdot 10^7\text{Bq}$$

$$N_T = \frac{50\text{g}}{59\text{g/mol}} \cdot N_A = \frac{50}{59}\text{mol} \cdot 6.022 \cdot 10^{23}\text{atoms/mol} = 5.10 \cdot 10^{23}\text{atoms}$$

$$A = \lambda N = \Phi N_T \left(1 - e^{-\lambda t}\right)$$

$$3.7 \cdot 10^7\text{Bq} = 10^9\text{cm}^{-2}\text{s}^{-1} \cdot 37 \cdot 10^{-24}\text{cm}^2 \cdot 5.10 \cdot 10^{23}\text{atoms} \cdot \left(1 - e^{-4.17 \cdot 10^{-9}\text{s}^{-1} \cdot t}\right)$$

$$t = \frac{ln\left(1 - \frac{3.7 \cdot 10^7\text{Bq}}{10^9\text{cm}^{-2}\text{s}^{-1} \cdot 37 \cdot 10^{-24}\text{cm}^2 \cdot 5.10 \cdot 10^{23}\text{atoms}}\right)}{-4.17 \cdot 10^{-9}\text{s}^{-1} \cdot 60^2\text{s/h} \cdot 24\text{h/d}}$$

$$t = 5.44\text{d}$$

(b) Estimate the number of ^{59}Co atoms consumed in 1 week.
^{59}Co consumed $= \Phi N_T t = 10^9\text{cm}^{-2}\text{s}^{-1} \cdot 37 \cdot 10^{-24}\text{cm}^2 \cdot 5.10 \cdot 10^{23}\text{atoms} \cdot 7 \cdot 24 \cdot 60^2\text{s}$
^{59}Co consumed $= 1.14 \cdot 10^{16}$