Homework 3

Nuclear Engineering 162 Solutions

Due 20 February 2008

IV.1 Turner 8.12 (Page 202)
Derive Equation (8.12) from Equations (8.9)-(8.11).
Using Equation 8.9:

$$hv + mc^2 = hv' + E'$$

Equation 8.10:

$$\frac{hv}{c} = \frac{hv'}{c}\cos\theta + P'\cos\varphi$$

Equation 8.11:

$$\frac{hv'}{c}\sin\theta = P'\sin\varphi$$

Derive Equation 8.12:

$$hv' = \frac{hv}{1 + (hv/mc^2)(1-\cos\theta)}$$

Adding the squares of Equations 8.10 and 8.12 to eliminate P' and φ:

$$hv - hv'\cos\theta = cP'\cos\varphi$$

$$hv'\sin\theta - cP'\sin\varphi$$

$$(hv - hv'\cos\theta)^2 + (hv'\sin\theta)^2 = (cP')^2 \longrightarrow$$

By definition of the electron energy and momentum,

$$(E')^2 = (P')^2c^2 + m^2c^4$$

$$(P'c)^2 = hv' + m^2c^4$$

Substituting this equation back into the expression for $(1-\cos\theta)$ and multiplying by (hv/mc^2):

$$\frac{hv}{mc^2}\frac{mc^2(hv - hv')}{hvhv'} = \frac{hv - hv'}{hv'} = \frac{hv}{hv'} - 1$$

$$hv' = \frac{hv}{\frac{hv}{hv'}} = \frac{hv}{1 + \frac{hv}{mc^2}\frac{mc^2(hv-hv')}{hvhv'}} = \frac{hv}{1 + (hv/mc^2)(1-\cos\theta)}$$

III.2 Turner 8.16 (Page 203)
A 662-keV photon is Compton scattered at an angle of 120^o with respect to its incident direction.
(a) What is the energy of the scattered electron?

$$hv' = \frac{hv}{1 + \frac{hv}{mc^2}(1-\cos\theta)}$$

$$= \frac{662\text{keV}}{1 + \frac{662\text{keV}}{511\text{keV}}(1 - \cos 120^o)}$$

$$E' = hv - hv' + mc^2$$

$$T = hv - hv' = 662\text{keV} - 224.9\text{kev} = 437.1\text{keV}$$

(b) What is the angle between the paths of the scattered electron and photon?

$$P'c = \sqrt{E'^2 - m^2c^4}$$

$$E' = T + mc^2 = 437.1\text{keV} + 511\text{keV} = 948.1\text{keV}$$

$$P'c = \sqrt{(948\text{keV})^2 - (511\text{keV})^2} = 798.6\text{keV}$$

$$hv' \sin\theta - P'c \sin\varphi$$

$$\varphi = \sin^{-1}\left(\frac{hv' \sin\theta}{P'c}\right)$$

$$\varphi = \sin^{-1}\left(\frac{224.9\text{keV} \sin 120^o}{798.6\text{keV}}\right)$$

$$\varphi = 14.1^o$$

$$\angle = \varphi + \theta = 14.1^o + 120^o = 134.1^o$$

III.3 Turner 8.20 (Page 203)

In a Compton scattering experiment a photon is observed to be scattered at an angle of 122^o while the electron recoils at an angle of 17^o with respect to the incident photon direction.
(a) What is the incident photon energy?

$$\frac{\sin\theta}{sin\varphi} = \frac{P'c}{hv'}$$

$$\frac{hv}{hv'} = \cos\theta + \frac{P'c}{hv'}\cos\varphi = \cos\theta + \frac{\sin\theta}{\sin\varphi}\cos\varphi$$

$$= 1 + \frac{hv}{mc^2}(1 - \cos\theta)$$

$$hv = \left(\cos\theta + \frac{\sin\theta}{\sin\varphi}\cos\varphi - 1\right) \cdot \frac{mc^2}{1 - \cos\theta}$$

$$= \left(\cos 122^o + \frac{\sin 122^o}{\sin 17^o}\cos 17^o - 1\right) \cdot \frac{511\text{keV}}{1 - \cos 122^o}$$

$$= 415.4\text{keV}$$

(b) What is the frequency of the scattered photon?

$$hv' = \frac{hv}{1 + \frac{hv}{mc^2} \cdot (1 - \cos\theta)}$$

$$= \frac{415.5\text{keV}}{1 + \frac{415.5\text{keV}}{511\text{keV}} \cdot (1 - \cos 122^o)}$$

$$= 185.2\text{keV}$$

$$v' = \frac{185.2\text{keV}}{h} = \frac{185.2\text{keV}}{4.136 \cdot 10^{-15}\text{eV} \cdot \text{s}}$$

$$= 4.478 \cdot 10^{19}\text{s}^{-1}$$

(c) How much energy does the electron receIIIe?

$$T = 415.5\text{keV} - 185.2\text{keV} = 230.2\text{keV}$$

(d) What is the recoil momentum of the electron?

$$E' = T + mc^2 = 230.3\text{keV} + 511\text{keV} = 741.3\text{keV}$$
$$p'_c = \sqrt{E'^2 - m^2c^4}$$
$$= \sqrt{(741.3\text{keV})^2 - (511\text{keV})^2}$$
$$= 537.0\text{keV}$$
$$p' = \frac{537.0\text{keV}}{c} = 2.87 \cdot 10^{-22}\text{kg} \cdot \text{m} \cdot \text{s}^{-1}$$

III.4 Turner 8.25 (Page 204)

The Klein-Nishina cross section for the collision of a 1-MeV photon with an electron is $2.11 \cdot 10^{-25}\text{cm}^2$. Calculate, for Compton scattering on aluminum,

(a) the energy-transfer cross section (per electron cm^{-2})

From Table 8.1,

$$\frac{T_avg}{hv} = 0.440$$
$$\sigma_{ET} = \frac{T_{avg}}{hv}\sigma$$
$$\sigma_{ET} = 0.440 \cdot 2.11 \cdot 10^{-25}\text{cm}^{-2}$$
$$\sigma_{ET} = 9.28 \cdot 10^{-26}\text{cm}^{-2}$$

(b) the energy-scattering cross section (per electron cm^{-2})

$$\sigma_S = \sigma - \sigma_{ET}$$
$$= 2.11 \cdot 10^{-25}\text{cm}^{-2} - 9.28 \cdot 10^{-25}\text{cm}^{-2}$$
$$= 1.18 \cdot 10^{-25}\text{cm}^{-2}$$

(c) the atomic cross section

$$\sigma_e = Z_e\sigma$$
$$= 13 \cdot 2.11 \cdot 10^{-25}\text{cm}^2$$
$$= 2.74 \cdot 10^{-24}\text{cm}^2$$

(d) the linear attenuation coefficient.

$$\sigma = \frac{N_A \cdot \rho}{A} \cdot \sigma_e$$
$$= \frac{6.022 \cdot 10^{23}\text{atoms/mol} \cdot 2.70\text{g/cm}^3}{27\text{g/mol}} \cdot 2.74 \cdot 10^{-24}\text{cm}^2/\text{atom}$$
$$= 0.165\text{cm}^{-1}$$

III.5 Turner 8.35 (Page 205)

A narrow beam of 400-keV photons is incident normally on a 2 mm iron liner.
From Figure 8.8 and the Periodic Table on p556-557
For photon energy of 0.4MeV

For Fe:

$$\frac{\mu}{\rho}(\text{Fe}) = 0.092\text{cm}^2\text{g}^{-1}$$

$$\rho_{\text{Fe}} = 7.86\text{g/cm}^3$$

For Al:

$$\frac{\mu}{\rho}(\text{Al}) = 0.092\text{cm}^2\text{g}^{-1}$$

$$\rho_{\text{Al}} = 2.7\text{g/cm}^3$$

For Pb:

$$\frac{\mu}{\rho}(\text{Pb}) = 0.21\text{cm}^2\text{g}^{-1}$$

$$\rho_{\text{Pb}} = 11.4\text{g/cm}^3$$

(a) What fraction of the photons have an interaction in the liner?

$$N(x) = N_0 e^{-\mu x}$$

The fraction transmitted through a distance x with no interaction is gIIIen by $\frac{N(x)}{N_0}$, therefore the fraction that interacts in a gIIIen distance x is gIIIen by $1 - \frac{N(x)}{N_0}$.

$$1 - \frac{N(x)}{N_0} = 1 - e^{-\mu x} = 1 - e^{-\frac{\mu}{\rho}\rho x} = 1 - e^{-\left(0.092\text{cm}^2\text{g}^{-1}\right)\left(7.86\text{g/cm}^3\right)\cdot 0.2\text{cm}}$$
$$= 0.135$$

(b) What thickness of Fe is needed to reduce the fraction of photons that are transmitted without interaction to 10%

$$\frac{N(x)}{N_0} = e^{-\mu x} = 0.10$$

$$ln\left(\frac{N(x)}{N_0}\right) = -\mu x$$

$$x = \frac{ln\left(\frac{N(x)}{N_0}\right)}{\mu} = \frac{ln(10)}{\left(0.092\text{cm}^2\text{g}^{-1}\right)\left(7.86\text{g/cm}^3\right)}$$
$$x = 3.175\text{cm}$$

(c) If Al were used instead of Fe, what thickness would be needed in (b)?

$$x = \frac{ln\left(\frac{N(x)}{N_0}\right)}{\mu} = \frac{ln(10)}{\left(0.092\text{cm}^2\text{g}^{-1}\right)\left(2.7\text{g/cm}^3\right)}$$
$$x = 9.27\text{cm}$$

(d) How do the answers in (b) and (c) compare when expressed in g cm^{-2}?

$$x_{Fe} = 3.175\text{cm}$$

$$x_{Al} = 9.27\text{cm}$$

$$x_{Fe} \cdot \rho_{Fe} = 3.175\text{cm} \cdot 7.86\text{g/cm}^3 = 24.95\text{gcm}^{-2}$$

$$x_{Al} \cdot \rho_{Al} = 9.27\text{cm} \cdot 2.7\text{g/cm}^3 = 25.02\text{gcm}^{-2}$$

$$x_{Fe} \cdot \rho_{Fe} \approx x_{Al} \cdot \rho_{Al} \approx 25\text{gcm}^{-2}$$

(e) If lead were used in (b), how would its thickness in g cm^{-2} compare with those for Al and Fe?

$$x = \frac{ln\left(\frac{N(x)}{N_0}\right)}{\mu} = \frac{ln\left(\frac{N(x)}{N_0}\right)}{\frac{\mu}{\rho} \cdot \rho}$$

$$x \cdot \rho_{Pb} = \frac{ln\left(\frac{N(x)}{N_0}\right)}{\frac{\mu}{\rho}} = \frac{ln(10)}{0.21\text{cm}^2\text{g}^{-1}}$$

$$x \cdot \rho_{Pb} = 11.0\text{gcm}^{-2}$$

III.6 Turner 8.43 (Page 205)

A pencil beam of 200-keV photons is normally incident on a 1.4-cm-thick sheet of aluminum pressed against a 2-mm-thick sheet of lead behind it.

(a) What fraction of the incident photons penetrate both sheets without interacting?

Mass attenuation coefficients and densities for this problem are obtained from Radiological Assessment Sources and Exposures by Richard Faw and J. Kenneth Shultis.

For Al:

$$\frac{\mu}{\rho}(\text{Al}) = 0.1225\text{cm}^2\text{g}^{-1}$$

$$\rho_{\text{Al}} = 2.7\text{g/cm}^3$$

For Pb:

$$\frac{\mu}{\rho}(\text{Pb}) = 0.9913\text{cm}^2\text{g}^{-1}$$

$$\rho_{\text{Pb}} = 11.34\text{g/cm}^3$$

The fraction of the incident photons that do not interact in either the Al or Pb is gIIIen by:

$$\frac{N}{N_0} = e^{\left(-\left(\frac{\mu}{\rho}(\text{Al}) \cdot \rho_{\text{Al}} \cdot x_{\text{Al}}\right) - \left(\frac{\mu}{\rho}(\text{Al}) \cdot \rho_{\text{Pb}} \cdot x_{\text{Pb}}\right)\right)}$$

$$\frac{N}{N_0} = e^{\left(-0.1225\text{cm}^2\text{g}^{-1} \cdot 2.7\text{g/cm}^3 \cdot 1.4\text{cm} - 0.9913\text{cm}^2\text{g}^{-1} \cdot 11.34\text{g/cm}^3 \cdot 0.2\text{cm}\right)}$$

$$= 0.06645$$

(b) What would be the difference if the photons came from the other direction, entering to lead first and then the aluminum?

Since the question ask for the difference between unattenuated photons only, there is no difference between entering lead first and entering aluminum.

III.7 Turner 8.49 (Page 206)

The mass attenuation coefficient of Pb for 70-keV photons is 3.0 cm^2 g^{-1} and the mass energy–absorption coefficient is 2.9 cm^2 g^{-1} (Figs. 8.8 and 8.11)

(a) Why are these two values almost equal?

These two values are almost equal because the dominant interaction is photoelectric effect. In pair production, only an energy given by $1 - 2mc^2/hv$ is transfered. This transferred energy is comparatively less than that of photoelectric effect $1 - \delta/hv$ because only energy of the low energy photons from fluorescene radiation are not transferred in photoelectric effect. Also, the re-emitted photons are low energy in high Z material so they will have small g. Thus, if the dominant interaction is pair production (i.e. highly energtic photons), the difference between these two values will be much bigger.

(b) How many cm of Pb are needed to reduce the transmitted intensity of a 70-keV X-ray beam to 1% of its original value?

$$x = \frac{ln\left(\frac{N_0}{N}\right)}{\left(\frac{\mu}{\rho}\right)\cdot\rho} = \frac{ln\left(\frac{1}{0.01}\right)}{2.9\text{cm}^2/\text{g}\cdot 11.4\text{g/cm}^3}$$
$$= 0.139\text{cm}$$

(c) What percentage of the photons penetrate this thickness without interacting?

$$\frac{N}{N_0} = e^{\frac{\mu}{\rho}\cdot x} = e^{2.9\text{cm}^2/\text{g}\cdot 11.4\text{g/cm}^3\cdot 0.139\text{cm}}$$
$$= 0.86\%$$

III.8 Turner 8.51 (Page 207)
What fraction of the energy in a 10-keV X-ray beam is deposited in 5 mm of soft tissue?
From Figure 8.12
For soft tissue:

$$\frac{\mu}{\rho} = 5.3\text{cm}^2\text{g}$$
$$\rho_A = \rho_{H_2O} = 1\text{cm}^2/\text{g}$$
$$x = 0.5\text{cm}$$
$$1 - \frac{\psi}{\psi_0} = 1 - e^{-\mu x} = 1 - e^{-\left(5.3\frac{\text{cm}^2}{\text{g}}\right)\left(1\frac{\text{g}}{\text{cm}^3}\right)(0.5\text{cm})}$$
$$= 0.93$$

III.9 Turner 8.53 (Page 207)
A dentist places the window of a 100-kVp (100-kV, peak voltage) X-ray machine near the face of a patient to obtain an X ray of the teeth. Without filtration, considerable low-energy (assume 20 keV) X rays are incident on the skin.
(a) If the intervening tissue has a thickness of 5mm, calculate the fraction of the 20-keV intensity absorbed in it.
From Figure 8.11, Figure 8.12 and the Periodic Table (p556-557)
For soft tissue:
Photon energy= 10keV

$$\frac{\mu_e n}{\rho}(20\text{keV}) = 0.63\text{cm}^2/\text{g}$$
$$\rho = \rho_{H_20} = 1\text{g/cm}^3$$

For Al:

$$\frac{\mu_e n}{\rho}(20\text{keV}) = 2.8\text{cm}^2/\text{g}$$
$$\rho_{Al} = 2.7\text{g/cm}^3$$

$$1 - \frac{\psi}{\psi_0} = 1 - e^{-\mu x} = 1 - e^{\left(-\left(0.63\frac{\text{cm}^2}{\text{g}}\right)\left(1\frac{\text{g}}{\text{cm}^3}\right)(0.5\text{cm})\right)}$$
$$= 0.27$$

(b) What thickness of aluminum filter would reduce the 20-keV radiation exposure by a factor of 10?

$$\frac{\psi}{\psi_0} = 1 - e^{-\mu x} = \frac{1}{10}$$
$$x = \frac{ln(10)}{\mu_{en}} = \frac{ln(10)}{(2.8\text{cm}^2\text{g})\,(2.7\text{g/cm}^3)}$$
$$= 0.3\text{cm}$$

(c) Calculate the reduction in the intensity of 100-keV X rays transmitted by the filter.

$$1 - \frac{\psi}{\psi_0} = 1 - e^{-\mu x} = 1 - e^{\left(-\left(0.036\frac{\text{cm}^2}{\text{g}}\right)\left(2.7\frac{\text{g}}{\text{cm}^3}\right)(0.3\text{cm})\right)}$$
$$= 0.0287 = 2.87\%$$

(d) After adding the filter, the exposure time need not be increased to obtain the quality of X-ray picture. Why not?
The majority of 100keV x-rays are still transmitted through the filter (¿97%) while nearly all of the 20keV x-rays are absorbed. Since it is the 100keV x-rays which contribute to the image, the exposure time need not be increased.

III.10 Obtain the following data/plots from the internet:

(a) Photon interaction coefficients for H, C, N, O, Al, Fe, Pb
(b) Mass energy absorption coefficients of human tissues
(c) Material constants and composition for
TISSUE, SOFT (ICRU-44)
MUSCLE, SKELETAL (ICRU-44)
BONE, CORTICAL (ICRU-44)
http://physics.nist.gov/PhysRefData/XrayMassCoef/cover.html
(d) The mass attenuation and the mass energy-absorption coefficients for
TISSUE, SOFT (ICRU-44)
MUSCLE, SKELETAL (ICRU-44)
BONE, CORTICAL (ICRU-44)
http://physics.nist.gov/PhysRefData/XrayMassCoef/cover.html

III.11 Turner 7.9 (Page 171)
9. From Fig. 5.6 find LET_∞ for a
(a) 2-MeV alpha particle,

$$1.8 \cdot 10^3\text{MeV} \cdot \text{cm}^{-1}$$

(b) 100-MeV moon,

$$2.3\text{MeV} \cdot \text{cm}^{-1}$$

(c) 100-keV positron.

$$4.3\text{MeV} \cdot \text{cm}^{-1}$$

III.12 Turner 7.12 (Page 171)
In Section 7.4 we calculated the specific ionization of a 5-MeV alpha particle in air ($3.42 \cdot 10^4\text{cm}^{-1}$) and in water ($4.32 \cdot 10^7\text{cm}^{-1}$). Why is the specific ionization so much greater in water?

The stopping power in a medium is directly proportional to the number of electrons per unit volume in the medium and roughly inversely proportional to the ionization energy so the specific ionization in air for alpha particles is much greater than that in water.

III.13 Turner 7.14 (Page 171)
(a) Define energy straggling.
The phenomenon of unequal losses of particles in a medium under identical conditions is called energy straggling (See Page 161).

(b) Does energy straggling cause range straggling?
Yes.