Homework 1 Solutions

Nuclear Engineering 162

Due 30 January 2007

I.1 Turner 2.5 (Page 48) Estimate the number of atoms/cm^2 in an aluminum foil that is 1 mm thick.

$$\frac{\rho \cdot \text{thickness} \cdot \text{N}_a}{\text{AtomicWeight}} = \text{atoms/cm}^2$$

$$\frac{2.70\text{gm/cm}^3 \cdot 0.1\text{cm} \cdot 6.022 \cdot 10^{23}\text{mol}^{-1}}{26.96\text{gm/mol}} = 6.03 \cdot 10^{21}\text{atoms/cm}^2$$

I.2 Turner 2.8 (Page 48) What is the minimum distance to which the 7.69MeV alpha particles could approach the center of the gold nuclei in Rutherford's experiments?
Method 1

$$T = 7.69\text{MeV} = V$$

$$V = \frac{Z_1 Z_2 \text{e}^2}{r} \longrightarrow r = \frac{Z_1 Z_2 \text{e}^2}{V}$$

$$r = \frac{Z_1 Z_2 \text{e}^2}{V}\frac{\hbar\text{c}}{\hbar\text{c}} = \frac{2 \cdot 79}{7.69\text{MeV}}\frac{1}{137} \cdot 197\text{MeV} \cdot \text{fm}$$

$$r = 29.6\text{fm}$$

Method 2

$$E_{\alpha,k} = E_\text{c} = k_0 \frac{Z_1 Z_2 \text{e}^2}{r}[\text{J}]$$

$$r = k_0 \cdot \frac{Z_1 Z_2 \text{e}}{E_{\alpha,k}[\text{J}]} = \frac{9 \cdot 10^9\text{nm/C}^2 \cdot 2 \cdot 79 \cdot (1.6022 \cdot 10^{-19}\text{C})^2}{7.69\text{MeV} \cdot 1.6022 \cdot 10^{-13}\text{J/MeV}}$$

$$r = 2.96 \cdot 10^{-14}\text{m} = 29.6\text{fm}$$

I.3 Turner 2.9 (Page 48) How much energy would an alpha particle need in order to "just touch" the nuclear surface in a gold foil?

$$r = r_a + r_{Au}$$

$$r_a \approx 2.08\text{fm}$$

$$r_{Au} = r_0 * A^{1/3} = 1.3\text{fm} \cdot (197)^{1/3} = 7.56\text{fm}$$

$$r = 2.08\text{fm} + 7.56\text{fm} = 9.64\text{fm}$$

$$T_\alpha = V = \frac{Z_1 Z_2}{r}\frac{\text{e}^2}{4\pi\epsilon_0} = \frac{2 \cdot 79}{9.64\text{fm}} \cdot 1.43\text{MeV} \cdot \text{fm}$$

$$T_\alpha = 23.4\text{MeV}$$

I.4 Turner 2.15 (Page 49) Calculate the radius of the $n = 2$ electron orbit in the Bohr hydrogen atom.

$$r_n = \frac{n^2\hbar^2}{k_0 Z e^2 m} = 5.29 \cdot 10^{-11}\frac{n^2}{Z}\mathrm{m}$$
$$r_2 = 5.29 \cdot 10^{-11}\frac{4}{1}\mathrm{m} = 2.12 \cdot 10^{-10}\mathrm{m}$$

I.5 Turner 2.25 (Page 50) How much energy is needed to remove an electron from the $n = 5$ state of He^+?

$$E_n = -\frac{13.6Z^2}{n^2}\mathrm{eV} = -\frac{13.6 \cdot 4}{25}\mathrm{eV} = -2.176\mathrm{eV}$$
$$0\mathrm{eV} - (-2.176eV) = 2.176eV$$

I.6 Turner 2.29 (Page 50) The negative muon is an elementary particle with a charge equal to that of the electron and a mass 207 times as large. A proton can capture a negative muon to form a hydrogen-like "mesic" atom. (The muon was formerly call the mu meson.) For such a system, calculate
(a) the radius of the first Bohr orbit
For these calculations, we have to use the reduced mass of the system in the center of mass coordinates. We indeed cannot assume that the difference of the masses between the proton and the muon is as small as used to be the one between the proton and the electron, and therefore we cannot say that the proton is immobile.

$$m = \frac{m_\mu m_p}{m_\mu + m_p} = 207 \cdot \frac{m_e m_p}{207 \cdot m_e + m_p} = 207 \cdot \frac{0.511\mathrm{MeV/c} \cdot 938.28\mathrm{MeV/c}}{207 \cdot 0.511\mathrm{MeV/c} + 938.28\mathrm{MeV/c}} = 95.06\mathrm{MeV/c}$$

$$r_n = \frac{n^2\hbar^2}{Zm}\frac{4\pi\epsilon_0}{\mathrm{e}^2} = \frac{n^2\hbar^2}{Zm}\frac{4\pi\epsilon_0}{\mathrm{e}^2}\frac{c^2}{c^2} = \frac{n^2(\hbar c)^2}{Zmc^2}\frac{4\pi\epsilon_0}{\mathrm{e}^2}$$
$$r_n = \frac{(197.3\mathrm{MeV} \cdot \mathrm{fm})^2}{1.43\mathrm{MeV} \cdot \mathrm{fm}}\frac{1}{95.06\mathrm{MeV}} = 286\mathrm{fm}$$

(b) the ionization potential.

$$E_n = -\frac{Z^2 m e^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} = \frac{1}{2}\left(\frac{\mathrm{e}^2}{4\pi\epsilon_0}\right)^2\frac{mc^2}{(\hbar c)^2} = \frac{1}{2}\frac{(1.43\mathrm{MeV} \cdot \mathrm{fm})^2}{(197.3\mathrm{MeV} \cdot \mathrm{fm})^2} \cdot 95.06\mathrm{MeV/c}$$

$$E_n = 0.00249\mathrm{MeV} = 2.5\mathrm{keV}$$

I.7 What is the energy in eV of a photon with frequency of 2 GHz? (b) What is the energy of a proton that has the same momentum as a 1-MeV photon?
(a)

$$E = h\nu = 4.136 \cdot 10^{-21}\mathrm{MeV} - \mathrm{sec} \cdot 2 \cdot 10^9\mathrm{Hz} = 8.27 \cdot 10^{-12}\mathrm{MeV} = 8.27 \cdot 10^{-6}\mathrm{eV}$$

(b)

$$p = \frac{E}{c} = \frac{8.27 \cdot 10^{-6}\mathrm{eV}}{3.00 \cdot 10^8\mathrm{m/s}} = 2.76 \cdot 10^{-14}\mathrm{eV} - \mathrm{s/m}$$
$$E_{proton} = \frac{p^2}{2m} = \frac{(2.76 \cdot 10^{-14}\mathrm{eV} - \mathrm{s/m})^2}{29.42 \cdot 10^8\mathrm{eV}/c^2} = 7.26 \cdot 10^{-20}\mathrm{eV}$$

I.8 Turner 3.4 (Page 78) Calculate the energy released when a thermal neutron is absorbed by deuterium.

$${}^1_0\mathrm{n} + {}^2_1\mathrm{H} \longrightarrow {}^3_1\mathrm{H} + Q$$
$$Q = [m_n + m_D - m_T]\mathrm{c} = \Delta_n + \Delta_D - \Delta_T = 8.0714 + 13.1359 - 14.95\mathrm{MeV}$$
$$Q = 6.26\mathrm{MeV}$$

I.9 Turner 3.5 (Page 78) Calculate the total binding energy of the alpha particle.

$$BE = 2M_n + 2M_p - M_\alpha = 2\Delta_n + 2\Delta_p - \Delta_\alpha$$

$$BE = 2 \cdot 8.0714\text{MeV} + 2 \cdot 7.289\text{MeV} - 2.4248\text{MeV} = 28.3\text{MeV}$$

Note: This solution makes the assumption that the electron binding energy is negligible, allowing the use of atomic rather than nuclear masses.

I.10 Turner 3.9 (Page 79) The atomic weight of ^{32}P is 31.973910. What is the value of Δ in MeV?

$$\Delta = M - A = 31.97391 - 32 = -0.02609\text{amu} \cdot 931.5\frac{\text{MeV}}{\text{amu}}$$

$$\Delta = -24.302\text{MeV}$$