V. NUMERICAL SOLUTION OF THE TRANSPORT EQUATION

The neutron transport equation can be solved analytically only for highly idealized configurations. For most transport problems of practical interest we resort to approximations. These can be physical (for example, ignoring anisotropic scattering) or numerical (for example, using finite-differences for derivatives), or both. Our goal is to convert the transport equation into a system of coupled algebraic equation that can be solved on a computer. Thus, we need to discretize the transport equation using either ***discrete ordinates methods*** or ***function expansions***. We shall briefly discuss here some of the more common approximations, discussing one independent variable at a time.

Having in mind that the general time-dependent neutron transport equation has seven independent variables (three spatial, two angular, energy, and time), a possible number of unknowns after the discretization (i.e., the number of algebraic equations to solve) is given in Table V.1.

TABLE V.6. An Example of Number of Unknowns after Discretization

Variabl e	**Lower Limit**	**Upper Limit**
Spatial	100	100,000
Energy	2	100
Angular	1	16
Time	1	1000
Iterations	20	200
Variations	1	1000
TOTAL	**$4x10^3$**	**$3x10^6$**

The number of variations can be related to the search for a critical mass, the control rod movement, the changes in material compositions, or the temperature changes. As can be see from Table V.1, the total number of equations to solve could be very large, as well as the time to solve them. Thus, it is essential to choose the efficient numerical methods and algorithms that are at the same time very accurate and comutationally very efficient. In addition, in many cases it is necessary to use very fast computers (supercomputers) or to use multiple processors to sole a large-scale problems.

V.1. Time Discretization

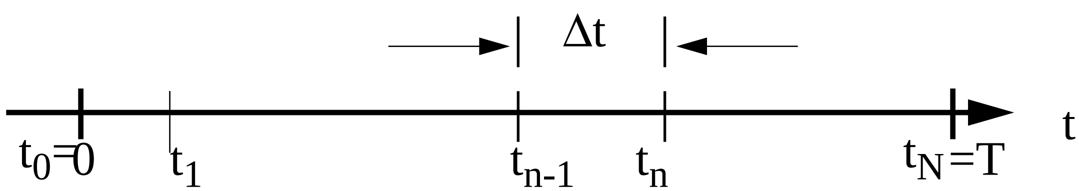
We start from the general time-dependent neutron transport equation (Eq. IV.23)

$$\frac{1}{v}\frac{\partial}{\partial t}\psi(\underline{r}, E, \underline{\Omega}, t) + \Sigma_t(\underline{r}, E)\psi(\underline{r}, E, \underline{\Omega}, t) + \underline{\Omega} \bullet \nabla\psi(\underline{r}, E, \underline{\Omega}, t) = \tag{V. 1}$$

$$\int_0^{\infty} \mathrm{dE'} \int_{4\pi} d\underline{\Omega}'\Sigma_s(\underline{r}, E' \rightarrow E, \underline{\Omega}' \bullet \underline{\Omega})\psi(\underline{r}, E', \underline{\Omega}', t) +$$

$$+ \frac{\chi(E)}{4\pi}\left(\int_0^{\infty} \mathrm{dE'}\nu(E')\Sigma_f(\underline{r}, E')\Phi(\underline{r}, E', t)\right) + \frac{1}{4\pi}Q_{ext}(\underline{r}, E, t)$$

We discretize the time interval (0, T) into N time steps



We first integrate Eq. V.1 from t = t_{n-1} to t = t_n, obtaining:

$$\frac{\psi_n - \psi_{n-1}}{v\Delta t} + \underline{\Omega} \bullet \nabla\overline{\psi}(\underline{r}, E, \underline{\Omega}) + \Sigma_t(\underline{r}, E)\overline{\psi}(\underline{r}, E, \underline{\Omega}) = \tag{V. 2}$$

$$\int_0^{\infty} \mathrm{dE'} \int_{4\pi} d\underline{\Omega}'\Sigma_s(\underline{r}, E' \rightarrow E, \underline{\Omega}' \bullet \underline{\Omega})\overline{\psi}(\underline{r}, E', \underline{\Omega}') +$$

$$+ \frac{\chi(E)}{4\pi}\left(\int_0^{\infty} \mathrm{dE'}\nu(E')\Sigma_f(\underline{r}, E')\overline{\Phi}(\underline{r}, E')\right) + \frac{1}{4\pi}\overline{Q}_{ext}(\underline{r}, E$$

where an overbar denotes an average over the time step, and where we have assumed that the cross sections do not change appreciably over the time step. For example, the angular flux averaged over the time step is defined as

$$\bar{\psi}(\underline{r}, E, \underline{\Omega}) = \frac{1}{\Delta t}\int_{t_{n-1}}^{t_n} \psi(\underline{r}, E, \underline{\Omega}, t)dt \qquad \text{(V. 3)}$$

where $\Delta t = t_n - t_{n-1}$. Also,

$$\int_{t_{n-1}}^{t_n} \frac{\partial}{\partial t}\psi(\underline{r}, E, \underline{\Omega}, t)dt = \psi(\underline{r}, E, \underline{\Omega}, t_n) - \psi(\underline{r}, E, \underline{\Omega}, t_{n-1}) = \psi_n(\underline{r}, E, \underline{\Omega}) - \psi_{n-1}(\underline{r}, E, \underline{\Omega})$$

Note that the "old" angular flux, ψ_{n-1}, is known – this is the initial condition for the current time step. This leaves two unknown quantities, $\bar{\psi}$ and ψ_n. Thus, we need another equation. A common choice is:

$$\bar{\psi} = \beta\psi_n + (1-\beta)\psi_{n-1} \qquad \text{(V. 4)}$$

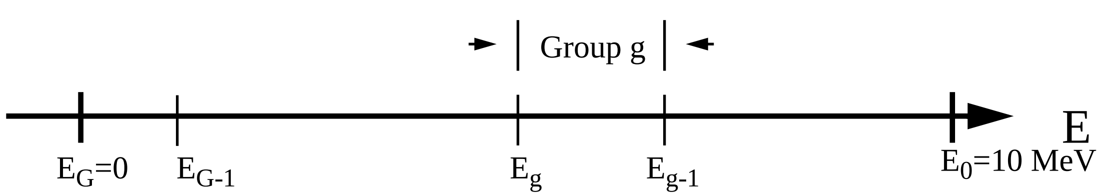
Using Eq. V.4 to eliminate ψ_n in Eq. V.2, we obtain:

$$\underline{\Omega} \bullet \nabla\bar{\psi} + \left(\Sigma_t + \frac{1}{v\beta\Delta t}\right)\bar{\psi} = \int_0^{\infty} dE' \int_{4\pi} d\underline{\Omega}'\Sigma_s\bar{\psi} + \frac{\chi(E)}{4\pi}\int_0^{\infty} dE'\nu\Sigma_f\bar{\Phi} + \frac{1}{4\pi}\bar{Q}_{ext} + \frac{\psi_{n-1}}{v\beta\Delta t} \qquad \text{(V. 5)}$$

Thus, the time-dependent problem reduces to a steady-state problem within each time step.

V.2. Energy Discretization

The neutrons in a reactor have energy range from about 10 MeV down to less than 0.01 eV. The energy variable is handled by the ***multigroup method***. The energy range of interest is divided into G subintervals called "groups":



We start with a steady state transport equation:

$$\underline{\Omega}\bullet\underline{\nabla}\psi(\underline{r},E,\underline{\Omega}) + \Sigma_t(\underline{r},E)\psi(\underline{r},E,\underline{\Omega}) = \frac{1}{4\pi}[\chi(E)]\int_{E_G}^{E_0} dE'\nu\Sigma_f(\underline{r},E')\phi(\underline{r},E')$$

$$+\int_{4\pi} d\Omega'\int_{E_G}^{E_0} dE'\Sigma_s(\underline{r},E'\to E,\underline{\Omega}'\to\underline{\Omega})\psi(\underline{r},E',\underline{\Omega}') + S(\underline{r},E,\underline{\Omega}) .$$

and integrate it over an arbitrary group g, obtaining:

$$\underline{\Omega}\bullet\underline{\nabla}\int_{E_g}^{E_{g-1}} dE\psi(\underline{r},E,\underline{\Omega}) + \frac{\int_{E_g}^{E_{g-1}} dE\Sigma_t(\underline{r},E)\psi(\underline{r},E,\underline{\Omega})}{\int_{E_g}^{E_{g-1}} dE\psi(\underline{r},E,\underline{\Omega})}\int_{E_g}^{E_{g-1}} dE\psi(\underline{r},E,\underline{\Omega})$$

$$= \sum_{g'=1}^{G}\int_{4\pi} d\Omega'\frac{\int_{E_{g'}}^{E_{g'-1}} dE'\left[\int_{E_g}^{E_{g-1}} dE\Sigma_s(\underline{r},E'\to E,\underline{\Omega}'\to\underline{\Omega})\right]\psi(\underline{r},E',\underline{\Omega}')}{\int_{E_{g'}}^{E_{g'-1}} dE'\psi(\underline{r},E',\underline{\Omega}')}\int_{E_{g'}}^{E_{g'-1}} dE'\psi(\underline{r},E',\underline{\Omega}')$$

$$+ \frac{1}{4\pi}\left[\int_{E_g}^{E_{g-1}} dE\chi(E)\right]\sum_{g'=1}^{G}\frac{\int_{E_{g'}}^{E_{g'-1}} dE'\nu\Sigma_f(\underline{r},E')\phi(\underline{r},E')}{\int_{E_{g'}}^{E_{g'-1}} dE'\phi(\underline{r},E')}\int_{E_{g'}}^{E_{g'-1}} dE'\phi(\underline{r},E') + \int_{E_g}^{E_{g-1}} dE\,S(\underline{r},E,\underline{\Omega}) .$$

where we replaced the integral over entire energy range by a sum of integrals over the particular energy groups:

$$\int_0^{E_0} dE' = \sum_{g'=1}^{G}\int_{E_{g'}}^{E_{g'-1}} dE'$$

We have not made any approximation — we have only integrated the transport equation. Next, we define the ***group fluxes***, ***group source***, and ***group fission spectrum*** as follows:

$$\psi_g(\underline{r}, \underline{\Omega}) = \int_{E_g}^{E_{g-1}} \psi(\underline{r}, E, \underline{\Omega}) dE \text{ , [neutrons/(cm}^2\text{ s sterad)]}$$

$$\phi_g(\underline{r}) = \int_{E_g}^{E_{g-1}} \phi(\underline{r}, E) dE \text{ , [neutrons/(cm}^2\text{ s)]}$$

$$S_g(\underline{r}, \underline{\Omega}) = \int_{E_g}^{E_{g-1}} S(\underline{r}, E, \underline{\Omega}) dE \text{ , [neutrons/(cm}^3\text{ s sterad)]}$$

$$\chi_g = \int_{E_g}^{E_{g-1}} \chi(E) dE \text{, [dimensionless].}$$

Now we make the ***multigroup approximation***. We assume that the neutron flux is of the separable form:

$$\psi(\underline{r}, E, \underline{\Omega}) \approx \varphi(\underline{r}, \underline{\Omega}) f(E) \qquad \text{(V. 6)}$$

where f(E) is a shape function in energy, often called a *spectrum*. With this assumption, we can define the **group cross sections** (group constants) as:

$$\frac{\int_{E_g}^{E_{g-1}} \Sigma_t(\underline{r}, E) \psi(\underline{r}, E, \underline{\Omega}) dE}{\int_{E_g}^{E_{g-1}} \psi(\underline{r}, E, \underline{\Omega}) dE} \rightarrow \frac{\int_{E_g}^{E_{g-1}} \Sigma_t(\underline{r}, E) f(E) dE}{\int_{E_g}^{E_{g-1}} f(E) dE} \equiv \Sigma_{t,g}(\underline{r})$$

$$\frac{\int_{E_g}^{E_{g-1}} \nu\Sigma_f(\underline{r}, E) \psi(\underline{r}, E, \underline{\Omega}) dE}{\int_{E_g}^{E_{g-1}} \psi(\underline{r}, E, \underline{\Omega}) dE} \rightarrow \frac{\int_{E_g}^{E_{g-1}} \nu\Sigma_f(\underline{r}, E) f(E) dE}{\int_{E_g}^{E_{g-1}} f(E) dE} \equiv \nu\Sigma_{f,g}(\underline{r})$$

$$\frac{\int_{E_{g'}}^{E_{g'-1}} dE' \int_{E_g}^{E_{g-1}} \Sigma_s(\underline{r}, E' \to E, \underline{\Omega}' \to \underline{\Omega})\psi(\underline{r}, E', \underline{\Omega}')dE}{\int_{E_{g'}}^{E_{g'-1}} \psi(\underline{r}, E', \underline{\Omega})dE'} \text{ --->}$$

$$\frac{\int_{E_{g'}}^{E_{g'-1}} dE' \int_{E_g}^{E_{g-1}} \Sigma_s(\underline{r}, E' \to E, \underline{\Omega}' \to \underline{\Omega}) f(E')dE}{\int_{E_{g'}}^{E_{g'-1}} f(E')dE'} \equiv \Sigma_{s,\, g' \to g}(\underline{r}, \underline{\Omega}' \to \underline{\Omega})$$

These definitions and approximations lead to the ***multigroup transport equation***:

$$\underline{\Omega} \bullet \nabla\psi_g(\underline{r}, \underline{\Omega}) + \Sigma_{t,\, g}(\underline{r})\psi(\underline{r}, \underline{\Omega}) = \sum_{g'=1}^{G} \int_{4\pi} d\underline{\Omega}'\Sigma_{s,\, g' \to g}(\underline{r}, \underline{\Omega}' \to \underline{\Omega})\psi_{g'}(\underline{r}, \underline{\Omega}') + \quad \text{(V. 7)}$$

$$+ \frac{\chi_g}{4\pi} \sum_{g'=1}^{G} \nu\Sigma_{f,\, g'}(\underline{r})\phi_{g'}(\underline{r}) + S_g(\underline{r}, \underline{\Omega})$$

Note: The multigroup transport equations are ***exact*** under ***either*** of the following conditions:

1) Angular flux is separable: $\psi(\underline{r}, E, \underline{\Omega}) \approx \varphi(\underline{r}, \underline{\Omega}) f(E)$, with known spectrum f(E).
2) Cross sections are constant:
   - $\Sigma_t(\underline{r},E)$ = constant for $E_g \le E \le E_{g-1}$,
   - $\Sigma_s(\underline{r},E' \to E,...)$ = constant for $E_{g'} \le E' \le E_{g'-1}$
   - $\nu\Sigma_f(\underline{r},E')$ = constant for $E_{g'} \le E' \le E_{g'-1}$.

The unknowns in the multigroup transport equation are the ***group fluxes***. These are ***not densities in energy***; each is an integral over an energy range.

Special Case: One Group

The one-group transport equation can be obtained by defining the angular flux in terms of delta function in energy : $\psi(\underline{r}, E, \underline{\Omega}) = \psi(\underline{r}, \underline{\Omega})\delta(E - E_0)$, and integrating the transport equation over energy range:

$$\underline{\Omega} \bullet \nabla\psi(\underline{r}, \underline{\Omega}) + \Sigma_t(\underline{r})\psi(\underline{r}, \underline{\Omega}) = \int_{4\pi} d\underline{\Omega}'\Sigma_s(\underline{r}, \underline{\Omega}' \bullet \underline{\Omega})\psi(\underline{r}, \underline{\Omega}') + \quad \text{(V. 8)}$$

$$+ \frac{1}{4\pi}\nu\Sigma_f(\underline{r})\phi(\underline{r}) + S(\underline{r}, \underline{\Omega})$$

This is often called the "one-speed" transport equation.

V.3.   Angle Discretization

The two most popular angular discretization methods are: the ***discrete-ordinates*** (**S_N**) and the ***spherical-harmonics*** (**P_N**) methods. Before we discuss these methods, however, we adopt the following assumption:

Assume: scattering cross sections depend only on the cosine of the scattering angle.

Thus,

$$\Sigma_s(\underline{r}, E'\rightarrow E, \underline{\Omega}'\rightarrow\underline{\Omega}) = \Sigma_s(\underline{r}, E'\rightarrow E, \underline{\Omega}'\bullet\underline{\Omega}) = S_s(\underline{r}, E'\rightarrow E, \mu_0) \ .$$

Let us look at the dependence of Σ_s on the scattering angle, whose cosine is μ_0. We almost always represent this dependence by a polynomial expansion. For reasons that will become clear later, we use ***Legendre polynomials***. Any reasonable function of some variable μ, with $-1\leq\mu\leq1$, can be represented as an infinite sum of Legendre polynomials:

$$f(\mu) = \sum_{k=0}^{\infty} \frac{2k+1}{4\pi} f_k P_k(\mu) \quad \text{(V. 9)}$$

where P_k is the k_{th} Legendre polynomial, and f_k is the k_{th} expansion coefficient, given by:

$$f_k = 2\pi\int_{-1}^{+1} d\mu P_k(\mu) f(\mu) \ .$$

We represent our scattering cross sections in this manner, except that we truncate the infinite sum at some finite order K:

$$\Sigma_s(\underline{r}, E' \to E, \mu_0) = \sum_{k=0}^{K} \frac{2k+1}{4\pi} \Sigma_{s,k}(\underline{r}, E' \to E) P_k(\mu_0) \tag{V. 10}$$

with

$$\Sigma_{s,k}(\underline{r}, E' \to E) \equiv \int_{4\pi} \Sigma_s(\underline{r}, E' \to E, \mu_0) P_k(\mu_0) d\underline{\Omega} = 2\pi \int_{-1}^{1} \Sigma_s(\underline{r}, E' \to E, \mu_0) P_k(\mu_0) d\mu_0$$

Note in particular:

$$\Sigma_{s,0}(\underline{r}, E' \to E) = \int_{4\pi} \Sigma_s(\underline{r}, E' \to E, \mu_0) d\underline{\Omega} = \Sigma_s(\underline{r}, E' \to E) \tag{V. 11}$$

$$\Sigma_{s,1}(\underline{r}, E' \to E) = 2\pi \int_{-1}^{1} \Sigma_s(\underline{r}, E' \to E, \mu_0) \mu_0 d\mu_0 \tag{V. 12}$$

The expression for $\Sigma_{s,1}(\underline{r},E'\to E)$ can be simplified by defining the mean scattering cosine, $\overline{\mu_0}$ as

$$\overline{\mu_0} = \frac{2\pi \int_{-1}^{1} \Sigma_s(\underline{r}, E' \to E, \mu_0) \mu_0 d\mu_0}{2\pi \int_{-1}^{1} \Sigma_s(\underline{r}, E' \to E, \mu_0) d\mu_0} \tag{V. 13}$$

Now, Eq. V.12 can be written as

$$\Sigma_{s,1}(\underline{r}, E' \to E) = \overline{\mu_0}\left(2\pi \int_{-1}^{1} \Sigma_s(\underline{r}, E' \to E, \mu_0) d\mu_0\right) = \overline{\mu_0} \Sigma_{s,0}(\underline{r}, E' \to E) \tag{V. 14}$$

Thus, for isotropic scattering (K = 0), we have

$$\Sigma_s(\underline{r}, E' \to E, \mu_0) = \frac{1}{4\pi} \Sigma_s(\underline{r}, E' \to E) \tag{V. 15}$$

and for linearly anisotropic scattering (K = 1) we obtain

$$\Sigma_s(\underline{r}, E' \to E, \mu_0) = \frac{1}{4\pi}[\Sigma_{s,0}(\underline{r}, E' \to E) + 3\mu_0(\Sigma_{s,1}(\underline{r}, E' \to E))] \quad \text{(V. 16)}$$

or

$$\Sigma_s(\underline{r}, E' \to E, \mu_0) = \frac{\Sigma_{s,0}(\underline{r}, E' \to E)}{4\pi}[1 + 3\overline{\mu_0}\mu_0] \quad \text{(V. 17)}$$

V.3.1 Spherical Harmonics (P_N) Method

The P_N method expands the angular flux in spherical harmonic functions, and truncates the expansion at order N:

$$\psi(\underline{r}, E, \underline{\Omega}) = \sum_{l=0}^{N} \sum_{m=-l}^{l} \phi_{lm}(\underline{r}, E) Y_{lm}(\underline{\Omega}) \quad \text{(V. 18)}$$

where the coefficients of expansion are given as

$$\phi_{lm}(\underline{r}, E) \equiv \int_{4\pi} \psi(\underline{r}, E, \underline{\Omega}) Y^{*}_{lm}(\underline{\Omega}) d\underline{\Omega} \quad \text{(V. 19)}$$

Here Y_{lm} is a spherical-harmonic function, and Y^{*}_{lm} is its complex conjugate (see D&H Appendix D), defined as

$$Y_{lm}(\underline{\Omega}) = (-1)^m \left[\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l(\mu) \exp(im\varphi) \quad \text{(V. 20)}$$

where $P_l(\mu)$ are Legendre polynomials, μ is the cosine of the polar angle and φ is the azimuthal angle. The spherical harmonic functions are orhogonal and normalized:

$$\int_{4\pi} Y^{*}_{lm}(\underline{\Omega}) Y_{l'm'}(\underline{\Omega}) d\underline{\Omega} = \delta_{l'l}\delta_{m'm} \quad \text{(V. 21)}$$

In the case of azimuthal symmetry, i.e., m = 0, the spherical harmonic functions depend on the cosine of the polar angle only:

$$Y_{l0}(\underline{\Omega}) = \left[\frac{2l+1}{4\pi}\right]^{1/2} P_l(\mu) \quad \text{(V. 22)}$$

Note that ϕ_0 is proportional to the scalar flux. With this approximation, we now have a set of unknown coefficients $\{\phi_{lm}\}$ to solve for instead of an unknown function of angle. To get equations for these coefficients, we can multiply the transport equation by an arbitrary spherical harmonic function, $Y^*_{l'm'}$, and integrate over all angles. We do this for all l' and m' such that $0 \le l' \le N$ and $-l' \le m' \le +l'$. This produces the same number of equations as unknowns. Now, we will look at the scattering term that arises when a spherical-harmonics expansion is used for the angular flux:

$$\int_{4\pi} d\Omega' \Sigma_s(\underline{r},E'\to E,\underline{\Omega}'\bullet\underline{\Omega})\psi(\underline{r},E',\underline{\Omega}')$$

$$\to \int_{4\pi} d\Omega' \left[\sum_{k=0}^{K} \frac{2k+1}{4\pi}\Sigma_{s,k}(\underline{r},E'\to E)P_k(\mu_0)\right]\left[\sum_{l=0}^{\infty}\sum_{m=-l}^{+l} \phi_{lm}(\underline{r},E')\, Y_{lm}(\underline{\Omega}')\right]$$

where μ_0 is $\underline{\Omega}\bullet\underline{\Omega}'$. Now, the spherical-harmonics *addition theorem* states that if μ_0 is the cosine of the angle between the directions $\underline{\Omega}$ and $\underline{\Omega}'$, then:

$$P_k(\mu_0) = \frac{4\pi}{2k+1}\sum_{n=-k}^{+k} Y^*_{kn}(\underline{\Omega}')Y_{kn}(\underline{\Omega})$$

This relationship allows us to simplify the scattering term. Once we substitute for $P_k(\mu_0)$, we can perform the integral over $d\underline{\Omega}'$ using the orthogonality of the spherical harmonics:

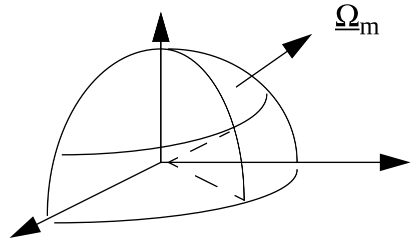
$$\int_{4\pi} d\Omega'\left[\sum_{k=0}^{K}\frac{2k+1}{4\pi}\Sigma_{s,k}(\ldots)\left(\frac{4\pi}{2k+1}\sum_{n=-k}^{k} Y^*_{kn}(\underline{\Omega}')Y_{kn}(\underline{\Omega})\right)\right]\left[\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\phi_{lm}(\ldots)\, Y_{lm}(\underline{\Omega}')\right]$$

$$= \sum_{k=0}^{K}\Sigma_{s,k}(\ldots)\sum_{n=-k}^{k} Y_{kn}(\underline{\Omega})\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\phi_{lm}(\ldots)\int_{4\pi} d\Omega'\, Y^*_{kn}(\underline{\Omega}')Y_{lm}(\underline{\Omega}')$$

$$= \sum_{k=0}^{K}\Sigma_{sk}(\underline{r},E'\to E)\sum_{n=-k}^{k}\phi_{kn}(\underline{r},E')Y_{kn}(\underline{\Omega}) \ .$$

V.3.2 Discrete-Ordinates (S_N) Method for Angle Discretization

The discrete-ordinates method, commonly referred to as the S_N method, assumes that neutrons travel only in ***discrete directions***.



Thus, each angular integral in the transport equation is replaced with a "***quadrature sum***":

$$\int_{4\pi} d\Omega f(\underline{\Omega}) \quad \underset{S_N}{\rightarrow} \quad \sum_{m=1}^{M} w_m f(\underline{\Omega}_m) \ .$$

The $\{w_m\}$ and $\{\underline{\Omega}_m\}$ constitute the **quadrature set.** Also, w_m corresponds to the surface segment on the unit sphere associated with the direction $\underline{\Omega}_m$. The discrete-ordinates method, then, makes the following approximation for the "***angular moments***" that appear in the scattering and fission terms:

$$\phi_{kn}(\underline{r},E) \quad \underset{S_N}{\rightarrow} \quad \sum_{m=1}^{M} w_m Y^*_{kn}(\underline{\Omega}_m)\psi(\underline{r},E,\underline{\Omega}_m) \ .$$

Thus, with the discrete-ordinates method we only need to find the angular flux at the M different angles $\{\underline{\Omega}_m\}$ in order to completely specify the total source. This means we only have to solve the transport equation along the M different angles $\{\underline{\Omega}_m\}$. If we define $\psi_{mg}=\psi_g(\underline{\Omega}_m)$, the multigroup discrete-ordinates equations can be written as:

$$\underline{\Omega}\bullet\underline{\nabla}\psi_{mg}(\underline{r}) + \Sigma_{t,g}(\underline{r})\psi_{mg}(\underline{r}) = \frac{\chi_g}{4\pi}\sum_{g'=1}^{G} \nu\Sigma_{f,g'}(\underline{r})\phi_{0,g'}(\underline{r})$$

$$+ \sum_{g'=1}^{G}\sum_{k=0}^{K} \Sigma_{sk,g'\rightarrow g}(\underline{r}) \sum_{n=-k}^{k} \phi_{kn,g'}(\underline{r})Y_{kn}(\underline{\Omega}_m) + S_{mg}(\underline{r}) \ , \quad m=1,...,M \ .$$

V.4. Spatial Discretization

There are many ways to discretize the spatial variable $\underline{\mathbf{r}}$ = (x,y,z). It depends on the geometry of the problem (one-, two-, or three-dimensional), and the coordinate system chosen (Cartesian or curvilinear). We will not discuss this lengthy topic.

V.5. Solution of the Discretized Equations: Iterative Methods

After discretizing all of the variables in the neutron transport, we obtain a large system of algebraic equations that can be solved on a computer. There are many numerical methods to solve the systems of algebraic equations, but the most popular are the iterative methods. This is another lengthy topic we will not cover in this course. The main steps in an iterative method are:

1) Guess the angular flux moments $\phi_{kn,g}$, thereby producing a guess for the total source.
2) For each group g and angle m, solve discretized equations for ψ_{mg}.
3) Use this ψ_{mg} to generate new flux moments, thereby producing a new guess for the total source.
4) Repeat (2) and (3) until convergence.

This is called source iteration, and it is used (usually in conjunction with some sort of iterative acceleration) in virtually all discrete-ordinates codes.