XV. Reactor Flux Measurements
(Special experiment at McClellan Nuclear Radiation Center)

The neutron spatial distribution and energy spectrum are some of the most important characteristics of a reactor. Spatial distribution is the subject of the subcritical assembly experiment planned for the last experiment of the semester. The neutron energy spectrum is the subject of an experiment to be done at the MNRC.

Neutron Energy Distribution

Almost all power reactors in use are thermal (moderated) reactors. A true thremal velocity spectrum would follow a Maxwell-Boltzmann distribution:

$$n(v) = \frac{dn}{dv} = n_0 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT} \tag{1}$$

In terms of energy, the distribution is

$$n(E) = \frac{dn}{dE} = n_0 \frac{2\pi}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT} \tag{2}$$

Note the following properties of the distribution:*

1. It is normalized to the total number of neutrons.

$$\int_0^\infty n(v)dv = \int_0^\infty n(E)dE = n_0 \tag{3}$$

2. The most probable neutron speed v_p is $\sqrt{\frac{2kT}{m}}$

3. The kinetic energy of a neutron traveling at the most probable speed is $\frac{1}{2} m \frac{2kT}{m} = kT$

4. The average neutron speed is

$$\langle v \rangle = \frac{1}{n_0} \int_0^\infty v n(v) dv = \sqrt{\frac{8kT}{\pi m}} \tag{4}$$

5. The most probable kinetic energy E_p is $\frac{1}{2} kT$.

6. The average neutron kinetic energy is

$$\langle E \rangle = \int_0^\infty E n(E) dE = \frac{3}{2} kT \tag{5}$$

Note that *n* is the **number** of neutrons; the neutron **flux** is *nv*.

There are a number of reasons why the actual neutron spectrum in a reactor deviates from a Maxwell-Boltzmann distribution. Principal among these are:

(1) Lower energy neutrons are preferentially absorbed by the moderator and other materials. This shifts the distribution to higher energies, or "hardens" the spectrum, as illustrated in fig. 1. Hardening is greatest in media whose capture cross-section is significant compared to the scattering cross-section.

(2) The fission neutrons have much higher than thermal energies. The slowing-down process tends to produce a spectrum of "epithermal" neutrons whose energy distribution follows a 1/E law, resulting in the high-energy tail in fig. 1.

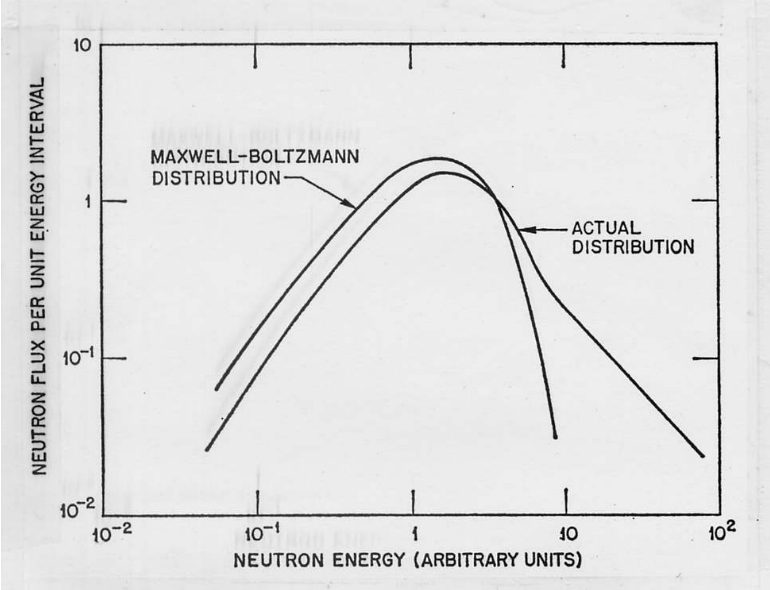

Fig. 1. Thermal and Epithermal neutron spectrum

(3) There is also a fast flux of unmoderated neutrons whose energy distribution is determined by the fission-neutron spectrum (fig. 2).

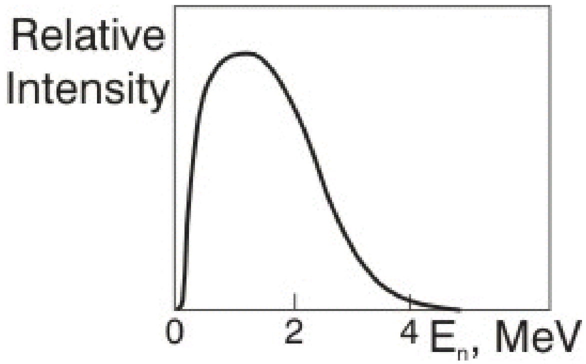

Fig. 2. Fission neutron spectrum

The ratio of fast to thermal flux depends on the type of reactor and the position within the reactor.

Effective Thermal Cross-Section

Microscopic cross-sections for isotopes are measured with and tabulated for **monoenergetic** neutrons. By convention, "thermal" cross-sections are measured at a neutron energy of 0.0253 eV, corresponding to a speed of 2200 m/s and a temperature of 293 K (20ºC). **This is different from the effective cross-section for neutrons with a Maxwell-Distribution at 293 K.** The latter depends on the energy dependence of the cross-section. For the many substances whose cross-section is proportional to 1/*v*, i.e.,

$$\sigma(v) = \sigma^{th} \frac{2200\ \mathrm{m/s}}{v} \tag{6}$$

the effective cross-section at temperature T is equal to the cross-section averaged over the neutron flux:

$$\sigma^{eff}(T) = \frac{\int_0^\infty \sigma^{th} \frac{2200\ \mathrm{m/s}}{v} n(v) v dv}{\int_0^\infty n(v) v dv} \tag{7}$$

$$= \frac{\sqrt{\pi}}{2}\left(\frac{293\ \mathrm{K}}{T}\right)^{1/2} \sigma^{th} = \frac{1}{1.128}\left(\frac{293\ \mathrm{K}}{T}\right)^{1/2} \sigma^{th}$$

(7a)

Note that, due to hardening, the effective temperature may be higher than that of the moderator. It is also worth noting that the factor 1.128 is equal to the ratio of the average neutron velocity to the most probable velocity. This ratio is thus independent of the equilibrium temperature.

Epithermal Cross-Sections

Capture of epithermal neutrons is usually expressed in terms of the **resonance integral**, an integrated cross-section defined as

$$\sigma^{RI} = \int \frac{\sigma(E)}{E} dE \tag{8}$$

where the integral extends over the epithermal energy region, typically 0.4 to 10^5 eV. The "resonance integral flux" Φ^{RI} is defined so that the capture rate per capturing atom per unit time is equal to $\Phi^{RI}\sigma^{RI}$. If the epithermal spectrum follows the 1/E law, the relationship of the total epithermal flux and Φ^{RI} is given by

$$\Phi^{epi} \equiv \int_{0.4\text{ eV}}^{10^5\text{ eV}} \phi(E)dE = \Phi^{RI} \ln\frac{10^5}{0.4} = 12.4\Phi^{RI} \tag{9}$$

* Equations 3-5 involve definite integrals of the form

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \tag{10}$$

This integral is solved by noting that

$$\int_0^\infty e^{-x^2} dx = \frac{1}{2}\int_{-\infty}^{\infty} e^{-x^2} dx = \frac{1}{2}\sqrt{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)} dxdy} \tag{11}$$

and transforming the double integral in (11) into polar coordinates.

Related integrals can be calculated from (10) by substitution and integration by parts:

$$\int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} \tag{12}$$

$$\int_0^\infty x^2 e^{-\alpha x^2} dx = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}} \tag{13}$$

$$\int_0^\infty x^{1/2} e^{-\alpha x} = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}} \tag{14}$$