

DEPARTMENT OF NUCLEAR ENGINEERING
University of California
Berkeley, California

NE 104a Experiment #9
(Revised 1/2007 by K. Vetter)

Neutron Flux Measurement
at the
McClellan Nuclear Radiation Center (MNRC)
Saturday April 14, 2007

This experiment is part of a field trip to the MNRC, sponsored by the Department of Energy's "Innovations in Nuclear Infrastructure and Education" program. Information about the experiments provided by Ben Liu of the MNRC is attached. The following are some background information and specifications for a report.

Background

Buildup of an activity by reactor irradiation

If a stable isotope that captures neutrons to form a radioisotope is placed in a reactor for a length of time t , the activity (becquerels) of the radioisotope at the end of the irradiation will be:

$$A = N_{target}^0 \Phi \sigma \frac{\lambda}{\lambda - \Phi \sigma} \left(e^{-\Phi \sigma t_{irrad.}} - e^{-\lambda t_{irrad.}} \right) \quad (1)$$

where A is the activity of the neutron capture product, N_{target}^0 the number of target atoms at the start of the irradiation, Φ is the reactor flux, σ is the capture cross-section of the target, λ is the decay constant of the radioisotope, and $t_{irrad.}$ is the length of the irradiation.

Note the following approximations:

- (a) If $\Phi \sigma t_{irrad.} \ll 1$ the exponential $e^{-\Phi \sigma t_{irrad.}} \approx 1$ (low burnup of the target). This is usually the case, except for long irradiation of high-cross-section materials at high flux.
- (b) If $\lambda \gg \Phi \sigma$, the fraction $\frac{\lambda}{\lambda - \Phi \sigma} \approx 1$, i.e. decay rate of the activity is faster than the burnup rate of the target. This is the case except for long-lived activities produced from targets with high cross-section at high flux.
- (c) If $\lambda t_{irrad.} \ll 1$ (irradiation for much less than a half-life), and approximation (a) is valid, the factor $(e^{-\Phi \sigma t_{irrad.}} - e^{-\lambda t_{irrad.}})$ in equation 1 is $\approx \lambda t_{irrad.}$

Gamma-ray counts from an irradiated sample

The number of counts of an irradiated sample is given by

$$C = Ae^{-\lambda t_{decay}} (1 - e^{-\lambda t_{meas.}}) B_{\gamma} \epsilon_{\gamma} / \lambda \quad (2)$$

where A is the activity at the end of the irradiation from equation (1), t_{decay} is the decay time between the end of the irradiation and the beginning of the measurement, $t_{meas.}$ is the duration of the counting measurement, B_{γ} is the gamma branching of the measured gamma ray, and ϵ_{γ} is the detector

efficiency for the measured gamma ray. Note that if $\lambda t \ll 1$ (measurement for much less than a half-life), C can be approximated:

$$(d) \quad C = Ae^{-\lambda t_{decay}} t_{meas.} B_{\gamma} \epsilon_{\gamma}$$

Neutron capture cross-section and resonance integral

According to equations (1) and (2) with approximations (a) and (b), the product of cross-section and flux is given by

$$\sigma\Phi = \frac{C\lambda e^{\lambda t_{decay}}}{N_{target}^0 (1 - e^{-\lambda t_{irrad.}})(1 - e^{-\lambda t_{meas.}}) B_{\gamma} \epsilon_{\gamma}} \quad (3)$$

Consider neutron capture in two energy ranges, “thermal” and “epithermal.” The total activity will be the sum of the activity produced by thermal and epithermal capture:

$$\sigma^{th}\Phi^{th} + \sigma^{epi}\Phi^{epi} = K \quad (4)$$

where K is the right-hand side of equation (3).

By activation of a second target, it is possible to determine both the thermal and the epithermal flux:

$$\Phi^{th} = \frac{K_1\sigma_2^{epi} - K_2\sigma_1^{epi}}{\sigma_1^{th}\sigma_2^{epi} - \sigma_2^{th}\sigma_1^{epi}} \quad (5)$$

$$\Phi^{epi} = \frac{K_2\sigma_1^{th} - K_1\sigma_2^{th}}{\sigma_1^{th}\sigma_2^{epi} - \sigma_2^{th}\sigma_1^{epi}} \quad (6)$$

(This method works best if one target has a large thermal and a small epithermal cross-section, the other a small thermal and a large epithermal cross-section.)

Definition of thermal and epithermal fluxes and cross-sections

Tabulated “thermal” cross-sections are for monoenergetic neutrons of velocity 2200 m/s, energy 0.0253 eV, corresponding to a temperature of 293 K ($E=kT$). Because the neutrons have a “thermal” distribution of energies, the correct cross-section is not precisely the 2200 m/s value. If the neutron energies follow a

Maxwell-Boltzmann distribution for temperature T , and the cross-sections exhibits $1/v_n$ dependence, it can be shown that the effective cross-section is the 2200 m/s value multiplied by $\frac{\sqrt{\pi}}{2} \sqrt{\frac{293}{T}} = \frac{1}{1.128} \sqrt{\frac{293}{T}}$.

The epithermal cross-section is defined in terms of the *resonance integral*:

$$\sigma^{RI} = \int \frac{\sigma(E)}{E} dE \quad (7)$$

where the integral is taken over “epithermal” energies, typically from 0.4 to 10^5 eV.

Tabulated values of the resonance integral are calculated from the cross-section measured as a function of energy using equation (7). The corresponding flux Φ^{RI} is defined such that the rate of epithermal capture equals $N_{target} \Phi^{RI} \sigma^{RI}$, i.e., equation (4) becomes

$$\sigma^{th} \Phi^{th} + \sigma^{RI} \Phi^{RI} = K \quad (8)$$

Note that Φ^{RI} is *not* the total flux in the interval 0.4 to 10^5 eV. If the flux at energy E is proportional to $1/E$, as is nearly the case in thermal reactors, the total flux is given by

$$\Phi^{epi} = \int_{0.4 \text{ eV}}^{10^5 \text{ eV}} \phi(E) dE = \Phi^{RI} \ln \frac{10^5}{0.4} = 12.4 \Phi^{RI} \quad (9)$$

The logarithmic factor in equation (9) is referred to as the neutron *lethargy*.

Report

Design your own report. Be sure to cover the following points:

- Description of the experiment (purpose and method)
- The results (measured data)
- Conclusions based on the data (calculated thermal, epithermal, and fast flux).
- Answers to the following questions:
 1. Derive equations (1), (2), (5), and (6).
 2. Show that approximations (a) and (b) are valid for all of the substances measured.
 3. For ^{38}Cl , the shortest-lived activity, how large an error (in percent) results from the use of approximation (c)? Approximation (d)?

Some useful websites:

<http://mnrc.ucdavis.edu>

<http://www.nuc.berkeley.edu/dept/Courses/NE-104A/Gammasearch.html>

<http://www.nndc.bnl.gov>

UC Davis/ MNRC Experimental Facilities/
Neutron Flux Measurements

H. Ben Liu, Ph.D.

UC Davis/ MNRC

Nuclear Specialist

E-mail : hbliu@mnrc.ucdavis.edu

MNRC-supplied information. Cross-section, abundance, and gamma emission data modified 3/28/2005 by Mike Lederer

Neutron Flux Measurements

Date: April 8, 2006

Purpose: To determine the average thermal (< 0.4 eV), epithermal (0.4 eV to 100 keV), and fast (> 100 keV) neutron fluxes in the UCD/MNRC's PTS rabbit.

UCD/ MNRC's Pneumatic Transfer System (PTS):

The system is designed to quickly transfer individual samples into and out of the reactor core. The samples are placed in a polyethylene holder, "rabbit". The rabbit travels through aluminum tubing between the receiver and the terminus at near reactor core centerline. A blower assembly moving air through the system moves/ pulls the rabbit between the receiver and the terminus. A "rabbit" can accommodate four 1.5 c.c. polyethylene sample vials. Group A will use samples 1-3, group B samples 4-6.

(Top)	void
	#3, #6
	#2, #5
(Bottom)	#1, #4

Sample #3: 10.8 mg $\text{Na}_2\text{CO}_3 \cdot \text{H}_2\text{O}$ powder

Sample #6: 9.8 mg $\text{Na}_2\text{CO}_3 \cdot \text{H}_2\text{O}$ powder

Sample #2: 111.9 mg Zr foil

Sample #5: 111.3 mg Zr foil

Sample #1: 84.2 mg Ni foil

Sample #4: 84.4 mg Ni foil

PTS Irradiation : 20 sec @ 1.5 MW

Simultaneous Counting : $\text{Na}_2\text{CO}_3 \cdot \text{H}_2\text{O}$ @ DET25 at 10 cm
Zr @ DET50 at 10 cm
Ni @ DET99 at 10 cm
(5 to 20 min counting...)

(n, gamma) Reactions :

Target Isotope	Isotope Abundance	σ_{th} (barns)	σ_{RI} (barns)	Product Isotope	Product $T_{1/2}$
Na-23	1.0000	0.530±0.005	0.311±0.010	Na-24	14.9590(12) hr
Ni-64	0.00926(1)	1.52±0.03	0.98±0.15	Ni-65	2.5172(3) hr
Zr-94	0.1738(4)	0.0499±0.0024*	0.23±0.01	Zr-95	64.02(5) d
Zr-96	0.0280(2)	0.0229±0.0010*	5.3±0.3	Zr-97	16.91(5) hr

$\sigma_{0.0253 \text{ eV, th}}$:

Neutron capture (n, γ) reaction cross sections for 0.0253 eV thermal neutrons with an average velocity of 2,200 m/sec. Effective σ_{th} at T (~50 °C) = $\sigma_{0.0253 \text{ eV, th}} * 1/ 1.128 * (298/ T)^{0.5}$.

σ_{RI} :

Neutrons above the cadmium cutoff energy, 0.4 eV, which induce (n, γ) reactions are usually called the epithermal or resonance neutrons. The resonance integral cross sections are defined by the expression $\sigma_{RI} = \int \sigma(E) dE/E$ (from 0.4 eV to ∞).

(n, p) Reactions :

Target Isotope	Isotope Abundance	σ_{avg} (millibarns)	Product Isotope	Product $T_{1/2}$
Ni-58	0.68007(9)	113±11	Co-58	70.86(7) d
Ni-60	0.26223(8)	2.1±0.2	Co-60	5.2714(5) yr

σ_{avg} : At neutron energies of 100 keV and above, the (n, p), (n, n'), etc. reactions must be considered. Below a particular threshold energy, all of these reactions have a cross section of zero. Above the threshold energy the cross section exhibits an energy dependence and σ_{avg} is the average cross section for fast reaction neutrons > 100 keV.

Radioactive Isotopes :

Isotope	$T_{1/2}$	Energy (keV)	Gamma Abundance (%)
Na-24	14.9590 hr	1368.63* 2754.03	100.00 99.944(4)

Co-58	70.82 d	810.77*	99.448(8)
Co-60	5.27 yr	1173.24* 1332.50	99.9736(7) 99.9856(4)
Ni-65	2.5172 `hr	366.27 1115.55 1481.84*	4.81(5) 15.43(9) 23.59(14)
Zr-95	64.02 d	724.20 756.73*	44.17(13) 54.46(10)
Zr-97	16.91 h	743.36*	93.06(16)

* **Gamma energy recommended for use in activity calculations.**

1. Na₂CO₃·H₂O

$$\begin{aligned}A_{o, \text{Na-24}} &= \phi \sigma N (1 - \exp^{-\lambda T_i}) \\ &= (\phi_{0.0253 \text{ eV, th}} * \sigma_{0.0253 \text{ eV, th}} + \phi_{\text{RI}} * \sigma_{\text{RI}}) N (1 - \exp^{-\lambda T_i}) \\ A_{o, \text{Na-24}} &= C / [\exp^{-\lambda T_d} * \text{BF} * \epsilon]\end{aligned}$$

A_o : isotope activity at the end of irradiation,
 N : total number of target nuclei,
 T_i : irradiation time,
 C : count rate registered,
 T_d : decay time after the irradiation,
 BF : branching factor or gamma abundance,
 ϵ : detector efficiency, function of energy.

In a typical thermal reactor core spectrum, $\phi_{\text{RI}} \approx 9\%$ of $\phi_{0.0253 \text{ eV, th}}$ and therefore, both $\phi_{0.0253 \text{ eV, th}}$ and ϕ_{RI} can be determined, especially $\phi_{0.0253 \text{ eV, th}}$. Beware that ϕ_{RI} is epithermal neutron flux per unit lethargy, i.e. total epithermal neutron flux divided by $\ln(10^5/0.4) = 12.4$

2. Zr

$$\begin{aligned}A_{o, \text{Zr-95}} &= (\phi_{0.0253 \text{ eV, th}} * \sigma_{0.0253 \text{ eV, th, Zr-94}} + \phi_{\text{RI}} * \sigma_{\text{RI, Zr-94}}) N (1 - \exp^{-\lambda T_i}) \\ &= C / [\exp^{-\lambda T_d} * \text{BF} * \epsilon] \\ &\text{(insufficient counts during this experiments)} \\ A_{o, \text{Zr-97}} &= (\phi_{0.0253 \text{ eV, th}} * \sigma_{0.0253 \text{ eV, th, Zr-96}} + \phi_{\text{RI}} * \sigma_{\text{RI, Zr-96}}) N (1 - \exp^{-\lambda T_i}) \\ &= C / [\exp^{-\lambda T_d} * \text{BF} * \epsilon]\end{aligned}$$

Solving simultaneous equations will have $\phi_{0.0253 \text{ eV, th}}$ and ϕ_{RI} determined. More importantly, solving the $A_{o, \text{Zr-97}}$ equation will determine ϕ_{RI} provided that $\phi_{\text{RI}} \approx 9\%$ of $\phi_{0.0253 \text{ eV, th}}$.

3. Ni

$$\begin{aligned}A_o &= \phi \sigma N (1 - \exp^{-\lambda T_i}) \\ &= (\phi_{\text{fast}} * \sigma_{\text{avg}}) N (1 - \exp^{-\lambda T_i}) \\ A_o &= C / [\exp^{-\lambda T_d} * \text{BF} * \epsilon]\end{aligned}$$

Average fast neutron flux can be determined.