Nuclear Engineering 281
Fall Semester 1994
Final Examination

THREE HOURS. TWO $8\frac{1}{2}'' \times 11''$ SHEETS OF NOTES ALLOWED.
PLASMA FORMULARY ALLOWED. OPEN TEXTBOOK (CHEN ONLY).
ANSWER ALL QUESTIONS.

1. An electron is present in a magnetic field system given by:

$$\begin{aligned} B_Z &= B_0(1+\alpha Z) \\ B_R &= B_0(-\alpha R/2) \end{aligned},$$

in cylindrical coordinates.

   a. Find an expression for the poloidal flux $\phi = RA_\theta$ for thsi field.

   b. Find an expression for the canonical angular momentum P_θ.

   c. The electron has an initial velocity of $v_\theta(0)$ and $v_Z(0)$ at $z(0) = 0$, $\theta(0) = 0$, and $r(0) = -v_\theta(0)/\Omega_c$. ($\Omega_c$ is negative for this case.) Evaluate the energy and canonical momentum For the case $B = 1.0$T, $\alpha = 1.0\text{m}^{-1}$, and $v_\theta(0) = v_z(0) = 10^6\text{m s}^{-1}$.

   d . Using conservation of H and P_θ, find the ponit where the elctron is mirrored back in the $-\hat{z}$ direction.

   e . Compare this result with that obtained from μ-conservation.

2. A maxwellian electron plasma with an initial distribution function given by:

$$f_0(\vec{x}, \vec{v}, t) = n_0\left(\frac{m}{2\pi T}\right)^{3/2} \exp\left(\frac{-mv^2}{2T}\right)$$

is subjected to an oscillating RF field with a frequency of 1.0 GHz and an amplitude of 100 kV cm^{-1} with the electric field in the $\hat{z}$ direction. This field is turned on at $t = 0$.

   a. Find the solution to the single-particle electron equation of motion for this case, in terms of the initial velocity v_0.

   b. Find the temperature of the plasma if the one-dimensional RMS thermal velocity equals the rf-induced peak velocity.

c. Sketch the expected electron distribution function as the electron population comes to equilibrium with the applied electric field.

3. Suppose that a fluid described by the MHD set of equations is found to have an MHD instability. Suppose also that there is some means to independently vary the adiabatic constant $\gamma = c_p/c_v$ for this fluid. Would you expect that increasing the value of γ increases or decreases the growth rate for this mode? Justify your result using the MHD energy principle.

4. A whistler wave has a dispersion relation given by:

$$\omega = k^2 c^2 |\Omega_{ce}|/\omega_{pe}^2$$

Plasma waves propagate from a point source (such as a lightning discharge) through the ionosphere ($n_e = 10^6 \text{cm}^{-3}$, $|\vec{B}| = 0.2$ gauss) for 500 km before reaching an observer.

   a. Find the group velocity at 10 KHz for this wave.

   b. Find the group velocity at 5 KHz for this wave.

   c. Will the noise from the lighning centered at 10 KHz reach the observer before or after the 5 KHz noise? What is the separation in time between these signals, at the observer ?

5. A Taylor state satisfies the equation

$$\vec{\nabla} \times \vec{B} = \lambda \vec{B}$$

In a region of space. The related quantity, helicity, is given by:

$$K = \int_V \vec{A} \cdot \vec{B} d^3x$$

Suppose that an experimental device is thought to contain a plasma which has formed into a Taylor state. Suppose that the total magnetic energy W_m was known, and the plasma current at one point of the boundary of the plasma were known. Could the helicity of the device be determined from these measurements? how ?

6. Resistive MHD effects can cause the formation of magnetic islands localized to rational surfaces ($q = m/n$, m, n integer) in a tokamak plasma. The poloidal geometry can be "unwrapped" to show these islands in striated layers in (x,y) rectangular coordinates.

   a. Draw a magnetic field line diagram for this "unwrapped" depiction of the magnetic fields in (x,y) space, assuming that the islands are well separated.

   b. Draw the field-line map for the same case when the islands are not well separated, i. e. there is "overlap".

   c. How do the qualitative features of this map compare to the features of the so-called "standard mapping":

$$u_{m+1} = u_m + v_m$$
$$v_{m+1} = v_m - 2\pi\epsilon^2 \sin(2\pi u_{m+1})$$