Midterm Exam #1

Solutions

#1 The velocity of the CM: $v_{CM} = \frac{v_{10}}{M+1}$ (1)

atom is initially at rest, $u_{20} = v_{CM}$ (2)

Momentum conservation in CM:

$u_{10} = Mu_{20}$ (before); $u_{1f} = Mu_{2f}$ (after) (3)

Total energy conservation in the collision in the CM frame is:

$$\frac{1}{2}u_{10}^2 + \frac{1}{2}Mu_{20}^2 = \frac{1}{2}u_{1f}^2 + \frac{1}{2}Mu_{2f}^2 + E^* \quad (4)$$

Eliminate u_{10} and u_{1f} from (4) using (3):

$$M^2u_{20}^2 + Mu_{20}^2 = M^2u_{2f}^2 + Mu_{2f}^2 + 2E^*$$

Solving for u_{2f} and using (2) $u_{2f}^2 = v_{CM}^2 - \frac{2E^*}{M(M+1)}$ (5)

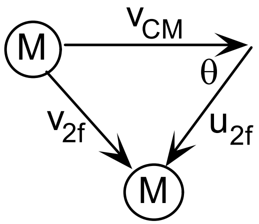
Write (5) as: $u_{2f} = v_{CM}\sqrt{B}$ (6)

Combine (5) & (6), using (1) and $v_{10}^2 = 2E_n$

$$B = 1 - \frac{2E^*}{M(M+1)v_{CM}^2} = 1 - \frac{2E^*(M+1)}{Mv_{10}^2} = 1 - \frac{E^*}{E_n}\frac{M+1}{M} = 1 - \frac{E_n^{th}}{E_n} \quad (7)$$

(20)

(b) The scattering diagram for the struck atom in the lab frame



Applying the law of cosines to this triangle and using (6) for $\mathbf{u_{2f}}$ gives:

$$v_{2f}^2 = v_{CM}^2 + u_{2f}^2 - 2v_{CM}u_{2f}\cos\theta = v_{CM}^2(1 + B - 2B\cos\theta)$$

Using $v_{2f}^2 = 2E/M$, and from (1), yields:

$$E = \frac{M}{(M+1)^2}E_n\left(1 + B - 2\sqrt{B}\cos\theta\right)$$

Eliminating B with (7) and noting that $\Lambda = 4M/(M+1)^2$ results in:

$$E = \frac{1}{4}\Lambda E_n\left(2 - \frac{E_n^{th}}{E_n} - 2\sqrt{1 - \frac{E_n^{th}}{E_n}}\cos\theta\right) = \frac{1}{2}\Lambda E_n\left(1 - \frac{1}{2}\frac{E_n^{th}}{E_n} - \sqrt{1 - \frac{E_n^{th}}{E_n}}\cos\theta\right)$$

#2 From Problem #1 of Prob. Set #2:

$$\sigma(E,\theta) = \frac{A}{E_{co}} \frac{\pi^2}{\sin\theta} \frac{\pi-\theta}{\theta^2 (2\pi-\theta)^2} \longrightarrow (\theta \to 0,\ \sin\theta \to \theta) \longrightarrow \sigma(E,\theta) = \frac{\pi A}{4E_{co}} \frac{1}{\theta^3}$$

$$E_{C0} = \frac{M_2}{M_1 + M_2} E \quad \textbf{(22)}$$

$$\sigma(E,T) = 2\pi\sigma[E,\theta(T)] \times [d(\cos\theta)/dT] \quad \textbf{(11a)}$$

$$T = \tfrac{1}{2}\Lambda E(1 - \cos\theta)$$

$(\theta \to 0)$

$(\cos\theta \to 1 - \theta^2/2)$

$$\theta \cong \sqrt{2}\sqrt{1-\cos\theta}$$

$$d(\cos\theta)/dT = \left(\tfrac{1}{2}\Lambda E\right)^{-1}$$

$$\sigma(E,T) = \frac{\pi^2}{4} A \sqrt{\frac{M_1}{M_2}} \frac{1}{E^{1/2}T^{3/2}}$$

$$T = \tfrac{1}{2}\Lambda E(1 - \cos\theta) \quad \textbf{(6)}$$

$$\theta \cong 2\sqrt{\frac{T}{\Lambda E}} \longrightarrow \sigma[E,\theta(T)] = \frac{\pi A}{4E} \frac{M_1 + M_2}{M_2} \frac{\Lambda^{3/2} E^{3/2}}{8T^{3/2}}$$

#3

Following the derivation of E_i for an interstitial and an edge dislocation,

$$E_i = -\sigma_h \Delta V = -4\pi\varepsilon\sigma_h$$

σ_h mean of the normal stresses (hydrostatic stress)

(a) screw dislocation - generates only a shear stresses, so $\sigma_h = 0$

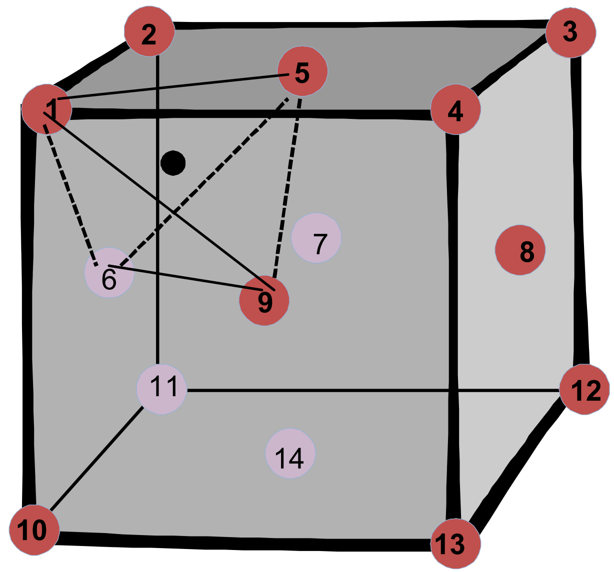
(b) Void: from Prob. 4 of the 3/18 class discussion,

$\sigma_{\phi\phi} = \sigma_{\theta\theta}$ and $\sigma_{\theta\theta} = -\frac{1}{2}\sigma_{rr}$. $\sigma_h = 1/3(\sigma_{\phi\phi} + \sigma_{\theta\theta} + \sigma_{rr}) = 0$

since $\sigma_h = 0$, $E_i = 0$, there is no force between an interstitial atom and a screw dislocation or a void

#5

(a) 1-5-6-9

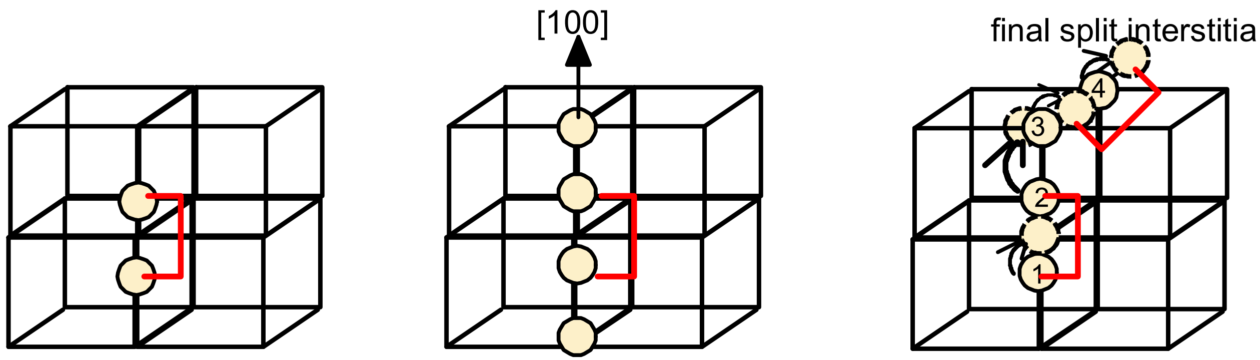


(b) interstitial sites are at the (¼ ¼ ¼) locations, so there are 8 of them. In the fcc lattice, there are 4 atoms, so there are 2 interstitial sites per atom

(c) The 8 interstitial sites form a cube whose side is ½ a_o

Therefore the interstitial jump distance is ½a_o

#6



(a) split interstitial

(b) crowdion

(c) interstitialcy

before

after